

- 1 a** $\sqrt{8} = \sqrt{4} \times \sqrt{2}$
 $= 2\sqrt{2}$
- b** $\sqrt{12} = \sqrt{4} \times \sqrt{3}$
 $= 2\sqrt{3}$
- c** $\sqrt{27} = \sqrt{9} \times \sqrt{3}$
 $= 3\sqrt{3}$
- d** $\sqrt{50} = \sqrt{25} \times \sqrt{2}$
 $= 5\sqrt{2}$
- e** $\sqrt{45} = \sqrt{9} \times \sqrt{5}$
 $= 3\sqrt{5}$
- f** $\sqrt{1210} = \sqrt{121} \times \sqrt{10}$
 $= 11\sqrt{10}$
- g** $\sqrt{98} = \sqrt{49} \times \sqrt{2}$
 $= 7\sqrt{2}$
- h** $\sqrt{108} = \sqrt{36} \times \sqrt{3}$
 $= 6\sqrt{3}$
- i** $\sqrt{25} = 5$
- j** $\sqrt{75} = \sqrt{25} \times \sqrt{3}$
 $= 5\sqrt{3}$
- k** $\sqrt{512} = \sqrt{256} \times \sqrt{2}$
 $= 16\sqrt{2}$
- 2 a** $\sqrt{8} + \sqrt{18} - 2\sqrt{2} = \sqrt{4 \times 2} + \sqrt{9 \times 2} - 2\sqrt{2}$
 $= 2\sqrt{2} + 3\sqrt{2} - 2\sqrt{2}$
 $= 3\sqrt{2}$
- b** $\sqrt{75} + 2\sqrt{12} - \sqrt{27} = \sqrt{25 \times 3} + 2\sqrt{4 \times 3} - \sqrt{9 \times 3}$
 $= 5\sqrt{3} + 4\sqrt{3} - 3\sqrt{3}$
 $= 6\sqrt{3}$
- c** $\sqrt{28} + \sqrt{175} - \sqrt{63} = \sqrt{4 \times 7} + \sqrt{25 \times 7} - \sqrt{9 \times 7}$
 $= 2\sqrt{7} + 5\sqrt{7} - 3\sqrt{7}$
 $= 4\sqrt{7}$
- d** $\sqrt{1000} - \sqrt{40} - \sqrt{90} = \sqrt{100 \times 10} - \sqrt{4 \times 10} - \sqrt{9 \times 10}$
 $= 10\sqrt{10} - 2\sqrt{10} - 3\sqrt{10}$
 $= 5\sqrt{10}$
- e** $\sqrt{512} + \sqrt{128} + \sqrt{32} = \sqrt{256 \times 2} + \sqrt{64 \times 2} + \sqrt{16 \times 2}$
 $= 16\sqrt{2} + 8\sqrt{2} + 4\sqrt{2}$
 $= 28\sqrt{2}$
- f** $\sqrt{24} - 3\sqrt{6} - \sqrt{216} + \sqrt{294} = \sqrt{4 \times 6} - 3\sqrt{6} - \sqrt{36 \times 6} + \sqrt{49 \times 6}$
 $= 2\sqrt{6} - 3\sqrt{6} - 6\sqrt{6} + 7\sqrt{6}$
 $= 0$

$$\begin{aligned}
 3 \text{ a } \quad \sqrt{75} + \sqrt{108} + \sqrt{14} &= \sqrt{25 \times 3} + \sqrt{36 \times 3} + \sqrt{14} \\
 &= 5\sqrt{3} + 6\sqrt{3} + \sqrt{14} \\
 &= 11\sqrt{3} + \sqrt{14}
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \quad \sqrt{847} - \sqrt{567} + \sqrt{63} &= \sqrt{121 \times 7} - \sqrt{81 \times 7} + \sqrt{9 \times 7} \\
 &= 11\sqrt{7} - 9\sqrt{7} + 3\sqrt{7} \\
 &= 5\sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \quad \sqrt{720} - \sqrt{245} - \sqrt{125} &= \sqrt{144 \times 5} - \sqrt{49 \times 5} - \sqrt{25 \times 5} \\
 &= 12\sqrt{5} - 7\sqrt{5} - 5\sqrt{5} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{d } \quad \sqrt{338} - \sqrt{288} + \sqrt{363} - \sqrt{300} &= \sqrt{169 \times 2} - \sqrt{144 \times 2} + \sqrt{121 \times 3} - \sqrt{100 \times 3} \\
 &= 13\sqrt{2} - 12\sqrt{2} + 11\sqrt{3} - 10\sqrt{3} \\
 &= \sqrt{2} + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{e } \quad \sqrt{12} + \sqrt{8} + \sqrt{18} + \sqrt{27} + \sqrt{300} &= \sqrt{4 \times 3} + \sqrt{4 \times 2} + \sqrt{9 \times 2} + \sqrt{9 \times 3} + \sqrt{100 \times 3} \\
 &= 2\sqrt{3} + 2\sqrt{2} + 3\sqrt{2} \\
 &\quad + 3\sqrt{3} + 10\sqrt{3} \\
 &= 5\sqrt{2} + 15\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{f } \quad 2\sqrt{18} + 3\sqrt{5} - \sqrt{50} + \sqrt{20} - \sqrt{80} &= 2\sqrt{9 \times 2} + 3\sqrt{5} - \sqrt{25 \times 2} + \sqrt{4 \times 5} - \sqrt{16 \times 5} \\
 &= 6\sqrt{2} + 3\sqrt{5} - 5\sqrt{2} + 2\sqrt{5} - 4\sqrt{5} \\
 &= \sqrt{2} + \sqrt{5}
 \end{aligned}$$

$$4 \text{ a } \quad \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\text{b } \quad \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

$$\text{c } \quad -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\text{d } \quad \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\text{e } \quad \frac{3}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$$

$$\text{f } \quad \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\begin{aligned}
 \text{g } \quad \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} &= \frac{\sqrt{2}-1}{2-1} \\
 &= \frac{\sqrt{2}-1}{1} \\
 &= \sqrt{2}-1
 \end{aligned}$$

$$\begin{aligned}
 \text{h } \quad \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} &= \frac{2+\sqrt{3}}{4-3} \\
 &= 2+\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{i } \quad \frac{1}{4-\sqrt{10}} \times \frac{4+\sqrt{10}}{4+\sqrt{10}} &= \frac{4+\sqrt{10}}{16-10} \\
 &= \frac{4+\sqrt{10}}{6}
 \end{aligned}$$

$$\begin{aligned} \text{j} \quad \frac{2}{\sqrt{6}+2} \times \frac{\sqrt{6}-2}{\sqrt{6}-2} &= \frac{2\sqrt{6}-4}{6-4} \\ &= \frac{2\sqrt{6}-4}{2} \\ &= \sqrt{6}-2 \end{aligned}$$

$$\begin{aligned} \text{k} \quad \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} &= \frac{\sqrt{5}+\sqrt{3}}{5-3} \\ &= \frac{\sqrt{5}+\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \text{l} \quad \frac{3}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} &= \frac{3(\sqrt{6}+\sqrt{5})}{6-5} \\ &= 3(\sqrt{6}+\sqrt{5}) \end{aligned}$$

$$\begin{aligned} \text{m} \quad \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} &= \frac{3+2\sqrt{2}}{9-8} \\ &= 3+2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{5 a} \quad \frac{2}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} &= \frac{6+4\sqrt{2}}{9-8} \\ &= 6+4\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{b} \quad (\sqrt{5}+2)^2 &= (\sqrt{5})^2 + 4\sqrt{5} + 4 \\ &= 5 + 4\sqrt{5} + 4 \\ &= 9 + 4\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{c} \quad (1+\sqrt{2})(3-2\sqrt{2}) &= 3 - 2\sqrt{2} + 3\sqrt{2} - 4 \\ &= -1 + \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{d} \quad (\sqrt{3}-1)^2 &= 3 - 2\sqrt{3} + 1 \\ &= 4 - 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{e} \quad \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{27}} &= \frac{1}{\sqrt{3}} \times \frac{\sqrt{27}}{\sqrt{27}} - \frac{1}{\sqrt{27}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{3\sqrt{3}-\sqrt{3}}{9} \\ &= \frac{2\sqrt{3}}{9} \end{aligned}$$

$$\begin{aligned} \text{f} \quad \frac{\sqrt{3}+2}{2\sqrt{3}-1} &= \frac{\sqrt{3}+2}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1} \\ &= \frac{6+\sqrt{3}+4\sqrt{3}+2}{12-1} \\ &= \frac{8+5\sqrt{3}}{11} \end{aligned}$$

$$\begin{aligned} \text{g} \quad \frac{\sqrt{5}+1}{\sqrt{5}-1} &= \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} \\ &= \frac{5+2\sqrt{5}+1}{5-1} \\ &= \frac{6+2\sqrt{5}}{4} \\ &= \frac{3+\sqrt{5}}{2} \end{aligned}$$

$$\begin{aligned}
 \text{h} \quad \frac{\sqrt{8} + 3}{\sqrt{18} + 2} &= \frac{2\sqrt{2} + 3}{3\sqrt{2} + 2} \\
 &= \frac{2\sqrt{2} + 3}{3\sqrt{2} + 2} \times \frac{3\sqrt{2} - 2}{3\sqrt{2} - 2} \\
 &= \frac{12 - 4\sqrt{2} + 9\sqrt{2} - 6}{18 - 4} \\
 &= \frac{6 + 5\sqrt{2}}{14}
 \end{aligned}$$

$$\begin{aligned}
 \text{6 a} \quad (2\sqrt{a} - 1)^2 &= (2\sqrt{a} - 1)(2\sqrt{a} - 1) \\
 &= 4a - 2\sqrt{a} - 2\sqrt{a} + 1 \\
 &= 4a - 4\sqrt{a} + 1
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad (\sqrt{x+1} + \sqrt{x+2})^2 &= (\sqrt{x+1} + \sqrt{x+2}) \times (\sqrt{x+1} + \sqrt{x+2}) \\
 &= x + 1 + 2\sqrt{(x+1)(x+2)} + x + 2 \\
 &= 2x + 3 + 2\sqrt{(x+1)(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 \text{7 a} \quad (5 - 3\sqrt{2}) - (6\sqrt{2} - 8) &= 5 - 3\sqrt{2} - 6\sqrt{2} + 8 \\
 &= 13 - 9\sqrt{2} \\
 &= \sqrt{169} - \sqrt{162} \\
 &> 0
 \end{aligned}$$

$5 - 3\sqrt{2}$ is larger.

$$\begin{aligned}
 \text{b} \quad (2\sqrt{6} - 3) - (7 - 2\sqrt{6}) &= 2\sqrt{6} - 3 - 7 + 2\sqrt{6} \\
 &= 4\sqrt{6} - 10 \\
 &= \sqrt{96} - \sqrt{100} \\
 &< 0
 \end{aligned}$$

$7 - 2\sqrt{6}$ is larger.

$$\text{8 a} \quad \frac{4}{3} < \frac{9}{2} \Rightarrow \frac{2}{\sqrt{3}} < \frac{3}{\sqrt{2}}$$

$$\text{b} \quad \frac{7}{9} < \frac{5}{4} \Rightarrow \frac{\sqrt{7}}{3} < \frac{\sqrt{5}}{2}$$

$$\text{c} \quad \frac{3}{49} < \frac{1}{5} \Rightarrow \frac{\sqrt{3}}{7} < \frac{\sqrt{5}}{5}$$

$$\text{d} \quad \frac{10}{4} < \frac{64}{3} \Rightarrow \frac{\sqrt{10}}{2} < \frac{8}{\sqrt{3}}$$

$$\begin{aligned}
 \text{9 a} \quad (x - \sqrt{3})(x + \sqrt{3}) &= x^2 - 3 \\
 \text{Therefore } b &= 0 \text{ and } c = -3
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad (x - 2\sqrt{3})(x + 2\sqrt{3}) &= x^2 - 12 \\
 \text{Therefore } b &= 0 \text{ and } c = -12
 \end{aligned}$$

$$\begin{aligned}
 \text{c} \quad (x - (1 - \sqrt{2}))(x - (1 + \sqrt{2})) &= x^2 - 2x - 1 \\
 \text{Therefore } b &= -2 \text{ and } c = -1
 \end{aligned}$$

$$\begin{aligned}
 \text{d} \quad (x - (2 - \sqrt{3}))(x - (2 + \sqrt{3})) &= x^2 - 4x + 1 \\
 \text{Therefore } b &= -4 \text{ and } c = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{e} \quad (x - (3 - 2\sqrt{2}))(x - (3 + 2\sqrt{2})) &= x^2 - 6x + 1 \\
 \text{Therefore } b &= -6 \text{ and } c = 1
 \end{aligned}$$

$$\text{f} \quad (x - (4 - 7\sqrt{5}))(x - (3 + 2\sqrt{5})) = x^2 - (-7 + 5\sqrt{5})x - 58 - 13\sqrt{5}$$

Therefore $b = -7 + 5\sqrt{5}$ and $c = -58 - 13\sqrt{5}$

$$\begin{aligned} 10 \quad \frac{1}{\sqrt{2} + \sqrt{3} + \sqrt{5}} \times \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{\sqrt{2} + \sqrt{3} - \sqrt{5}} &= \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{(\sqrt{2} + \sqrt{3})^2 - 5} \\ &= \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{(2 + 3 + 2\sqrt{6}) - 5} \\ &= \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{2\sqrt{6}} \\ &= \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{\sqrt{12} + \sqrt{18} - \sqrt{30}}{12} \\ &= \frac{2\sqrt{3} + 3\sqrt{2} - \sqrt{30}}{12} \end{aligned}$$

$$11 \quad \text{Note } a - b = \left(\frac{1}{a^3}\right)^3 - \left(\frac{1}{b^3}\right)^3$$

$$2 \quad \frac{1}{1 - 2\sqrt[3]{3}} \times \frac{1 + 2\sqrt[3]{3} + 2\sqrt[3]{9}}{1 + 2\sqrt[3]{3} + 2\sqrt[3]{9}} = -(1 + 2\sqrt[3]{3} + 2\sqrt[3]{9})$$